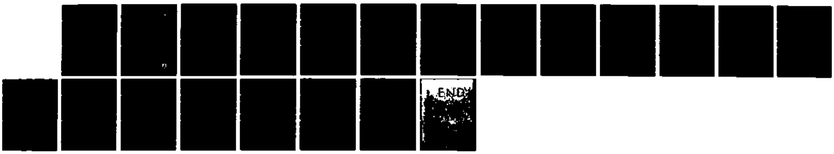
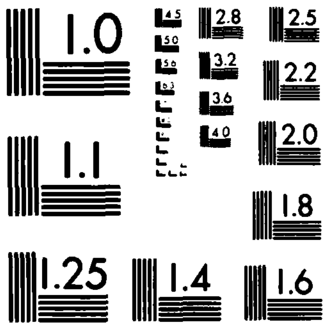
AD-A130 430 APPLICATION OF A DIRECT METHOD FOR REAL HEIGHT DETERMINATION TO TWO IONOGRAMS(U) AIR FORCE GEOPHYSICS LAB HANSCOM AFB MA M M KLEIN 01 MAR 83 AFGL-TR-83-0064 1/1

UNCLASSIFIED F/G 14/2 NL




1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6
6.3

AFGL-TR-83-0064
ENVIRONMENTAL RESEARCH PAPERS, NO. 827

Application of a Direct Method for Real Height Determination to Two Ionograms

MILTON M. KLEIN

1 March 1983

SPACE PHYSICS DIVISION PROJECT 2310
AIR FORCE GEOPHYSICS LABORATORY
HANSCOM AFB, MASSACHUSETTS 01731

AIR FORCE SYSTEMS COMMAND, USAF

This report has been reviewed by the ESD Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS).

This technical report has been reviewed and is approved for publication.

Alva T Stair, Jr.

DR. ALVA T. STAIR, Jr

Chief Scientist

Qualified requestors may obtain additional copies from the Defense Technical Information Center. All others should apply to the National Technical Information Service.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFGL-TR-83-0064	2. GOVT ACCESSION NO.: A130 430
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): APPLICATION OF A DIRECT METHOD FOR REAL HEIGHT DETERMINATION TO TWO IONOGRAMS	5. TYPE OF REPORT & PERIOD COVERED: Scientific. Interim.
	6. PERFORMING ORG. REPORT NUMBER: ERP, No. 827
7. AUTHOR(s): Milton M. Klein	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Air Force Geophysics Laboratory (PHY) Hanscom Air Force Base Massachusetts 01731	10. PROGRAM ELEMENT PROJECT TASK AREA & WORK UNIT NUMBERS: 61102F 2310G602
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Geophysics Laboratory (PHY) Hanscom Air Force Base Massachusetts 01731	12. REPORT DATE: 1 March 1983
	13. NUMBER OF PAGES: 17
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS (of this report): Unclassified
	15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Real height
Virtual height
Initial height
Ionosonde data
Ionogram
Electron density
Parametric form

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A direct method for determining real height from virtual height data has been applied to two ionograms from data recorded at Boulder, Colorado and Garchy, France. The initial height was obtained by a modification of the one-start procedure of Howe and McKinnis. The real height results obtained from the two ionograms were in good agreement with previously-reported accurate numerical calculations.

DD FORM 1473

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. Abstract (Contd)

To provide a simpler procedure for performing the required integration and thus make the present method more rapid and convenient, the virtual height data have been modeled in terms of a polynomial governed by a single parameter. The parameter was determined by comparison of the model with the data. The results using the model were in good agreement with those obtained by direct use of the data.

The present method affords a simple and convenient means of obtaining real height from virtual height data, within the accuracy of the experimental data.

Contents

Illustrations

Illustrations

Application of a Direct Method for Real Height Determination to Two Ionograms

1. INTRODUCTION

In a previous report,[1] a direct procedure was developed for the determination of real height from virtual height data obtained in the presence of a magnetic field. The method was based upon the representation of the dispersion relation between index of refraction and electron density by a simple power law, resulting in an Abel integral equation for the relation between real height and virtual height, which could then be solved explicitly for the real height as a function of frequency. To study the accuracy of the method, two parabolic models of electron density were converted into virtual height data, from which the real height was determined by the present method. The results obtained were in good agreement with the original models.

In the present report, the method has been extended to the analysis of field data. To assess the accuracy and reliability of the method, ionograms have been chosen for which accurate numerical calculations of real height were available for comparison.

(Received for publication 25 February 1983)

1. Klein, M.M. (1981) A Simplified Procedure for Direct Determination of Real Height From Virtual Height Data, AFGL-TR-81-0070, AD A104374.

2. APPLICATION OF THE PRESENT METHOD TO [illegible] DATA

The two ionograms chosen for analysis, corresponding to data obtained at Boulder, Colorado,[2] and Garchy, France,[3] are shown in Figures 1 and 2. Since the lowest frequency f_{min}, for which ionosonde data is available is usually limited to a range of 1 to 2 MHz, it is necessary to determine the initial height h_0, below which the effect of ionization may be considered negligible. To determine h_0 it is necessary to assume a variation in virtual height h' from f_{min} down to the frequency f_0 corresponding to the initial height. Since the Garchy ionogram is flatter in the lower frequency region than the Boulder curve, we may anticipate that the initial height calculation will be more accurate for Garchy than for Boulder.

As shown in Reference 1, the real height is obtained from the integral

$$h = \int_0^{f_v} p(\alpha) \frac{1}{f^{1-\alpha}} \frac{h'(f)}{(f_v^2 - f^2)^{\alpha/2}} \, df \quad , \tag{1}$$

where α is a frequency dependent parameter,

$$p(\alpha) = \frac{\sin\left(\pi \frac{\alpha}{2}\right)}{\pi \frac{\alpha}{2}} \tag{2}$$

and f_v is the maximum probing frequency. Since the lowest frequency for which data is available is f_0, we write Eq. (1) in the form

$$h - h_0 = \int_{f_0}^{f_v} p(\alpha) \frac{1}{f^{1-\alpha}} \frac{h'(f) - h_0}{(f_v^2 - f^2)^{\alpha/2}} \, df \quad , \tag{3}$$

so that the integral gives the increment in real height measured from h_0. A curve of the parameter α as a function of frequency, taken from Reference 1, is given in Figure 3.

---

2. Wright, J.W., et al (1972) Automatic N(h,t) profiles of the ionosphere with a digital ionosonde, Radio Sci. 11:1033-1043

3. Taieb, C. (1967) A quick model method for obtaining real-height parameters from routine ionosphere data, Radio Sci. 2(new series)(No. 10):1263-1267.

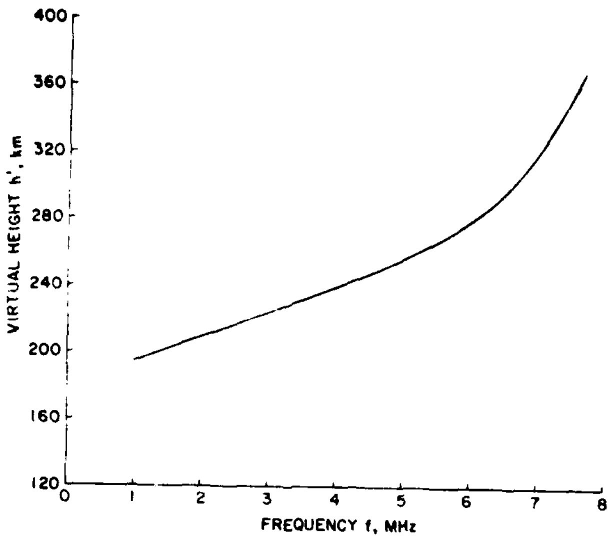


Figure 1. Boulder, Colorado, Ionogram, 20 February 1970, 1900:27 MST. Declination I = 67.5°

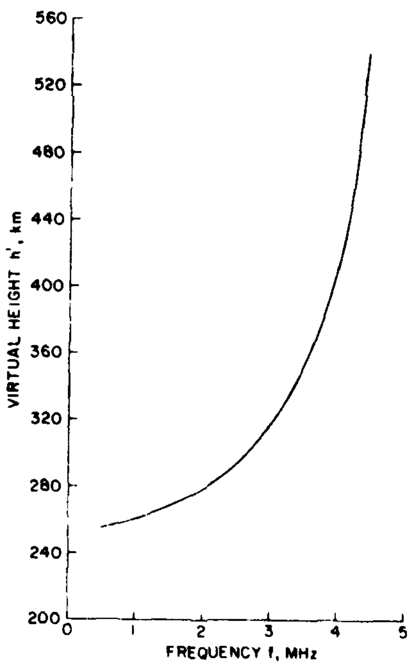


Figure 2. Garchy, France, Ionogram, 19 March 1961, 0200 Local Time. Declination I = 63°

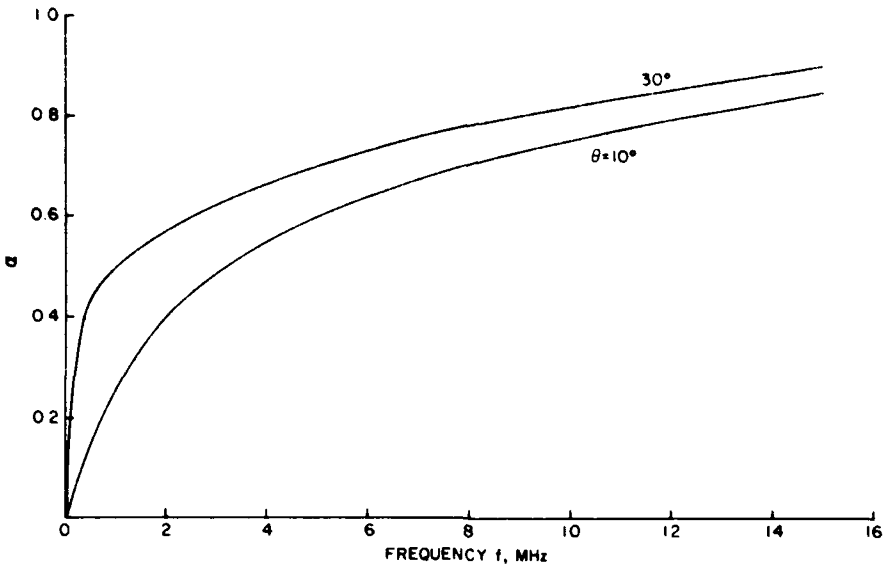


Figure 3. Plot of Parameter α Against Frequency; θ is Angle Between Ray and Magnetic Field

3. DETERMINATION OF INITIAL HEIGHT

We have found the one-start procedure of Howe and McKinnis[4] convenient to use and easy to adapt to our method. We write Eq. (3) in the form

$$\frac{h - h_0}{h' - h_0} = \int_{x_0}^{1} p(\alpha) \frac{h'(x) - h_0}{h'_v - h_0} \frac{1}{x^{1-\alpha}} \frac{dx}{(1 - x^2)^{\alpha/2}} , \tag{4}$$

where $x = f/f_v$ is a normalized frequency variable. We now assume a linear variation of h' between x_0 and nearby points x_1 and x_2 at which h' is known and write

$$\frac{h_1 - h_0}{h'_1 - h_0} = \int_{x_0}^{x_1} \frac{x - x_0}{x_1 - x_0} \frac{dx}{x} \tag{5}$$

---

4. Howe, H.H., and McInnis, D.E. (1967) Ionospheric electron-density profiles with continuous gradients and underlying ionization corrections. II. Formulation for a digital computer, Radio Sci. 2(New Series)(No. 10):1135-1158.

$$\frac{h_2 - h_0}{h'_2 - h_0} = \int_{x_0}^{x_2} \frac{x - x_0}{x_2 - x_0} \frac{dx}{x} \tag{6}$$

where, since α is small, we have taken $\alpha = 0$ and $p(\alpha) = 1$. Calculations show that changes in α for small α produce only slight changes in the integral. To determine h_0 we need an additional relation among the real heights h_0, h_1, and h_2 in Eqs. (5) and (6). We shall make the reasonable assumption that these real heights lie on a straight line and write

$$\frac{h_1 - h_0}{h_2 - h_0} = \frac{f_1 - f_0}{f_2 - f_0} \ , \tag{7}$$

From these equations we obtain

$$\frac{h'_2 - h_0}{h'_1 - h_0} = \frac{f_2 - f_0}{f_1 - f_0} \frac{I_1}{I_2} \ , \tag{8}$$

where

$$I_1 = \int_{x_0}^{x_1} \frac{x - x_0}{x_1 - x_0} \frac{dx}{x} = 1 - \frac{x_0}{x_1 - x_0} \ln \frac{x_1}{x_0} \tag{9}$$

$$I_2 = \int_{x_0}^{x_2} \frac{x - x_0}{x_2 - x_0} \frac{dx}{x} = 1 - \frac{x_0}{x_2 - x_0} \ln \frac{x_2}{x_0} \ . \tag{10}$$

Trial calculations showed the value of h_0 was slightly sensitive to the choice of x_0; a variation of 0.25 in x_0 yielded a change of about 3 km in h_0 for Boulder and less than one for Garchy. Since the calculated value of h_0 for Boulder was somewhat low, a parabolic scheme was tried in place of the preceding linear method. In place of Eqs. (5) and (6) we write

$$\frac{h_1 - h_0}{h_1' - h_0} = \int_{x_0}^{x_1} \left(\frac{x - x_0}{x_1 - x_0}\right)^2 \frac{dx}{x} \tag{11}$$

$$\frac{h_2 - h_0}{h_2' - h_0} = \int_{x_0}^{x_2} \left(\frac{x - x_0}{x_2 - x_0}\right)^2 \frac{dx}{x} \tag{12}$$

while Eqs. (8), (9), and (10) are replaced by

$$\frac{h_2' - h_0}{h_1' - h_0} = \frac{f_2 - f_0}{f_1 - f_0} \frac{J_1}{J_2} \tag{13}$$

$$J_1 = \int_{x_0}^{x_1} \left(\frac{x - x_0}{x_1 - x_0}\right)^2 = \frac{1}{2} \frac{x_1 - 3x_0}{x_1 - x_0} + \frac{x_0^2}{(x_1 - x_0)^2} \ln \frac{x_1}{x_0} \tag{14}$$

$$J_2 = \int_{x_0}^{x_2} \left(\frac{x - x_0}{x_2 - x_0}\right)^2 = \frac{1}{2} \frac{x_2 - 3x_0}{x_2 - x_0} + \frac{x_0^2}{(x_2 - x_0)^2} \ln \frac{x_2}{x_0} \quad . \tag{15}$$

The parabolic fit was found to substantially improve the initial height for the Boulder ionogram while leaving its value for the Garchy case virtually unchanged. We have, therefore, utilized the parabolic fit in the determination of initial height.

In the absence of data, the determination of h_0 by a method such as the present one, which relies on ordinary ray data only, would ignore any sudden changes in virtual height below the minimum frequency. If, however, data for the extraordinary ray is also used, sudden changes in the real height can be detected.[2,4] For example, the calculated value of real height for the Boulder case in Reference 2 shows a sudden rapid drop when the frequency decreases to about 0.85 MHz. Nevertheless, the value of h_0 obtained herein yields values of h at the higher frequencies consistent with those of Reference 2.

4. RESULTS

The calculated values of true height by the present method for the Boulder and Garchy ionograms are given in Figures 4 and 5. The calculated values of h_0 are the initial values for these curves. For comparison, the previously calculated values of true height for Boulder[2] and for Garchy[3] are also plotted in Figures 4 and 5. For both cases, the true height values by the present method are in good agreement with, and lying slightly below, the corresponding curves of References 2 and 3. The procedure presented herein accordingly appears to offer a simple method without undue computational labor for obtaining true height within the accuracy of the experimental data.

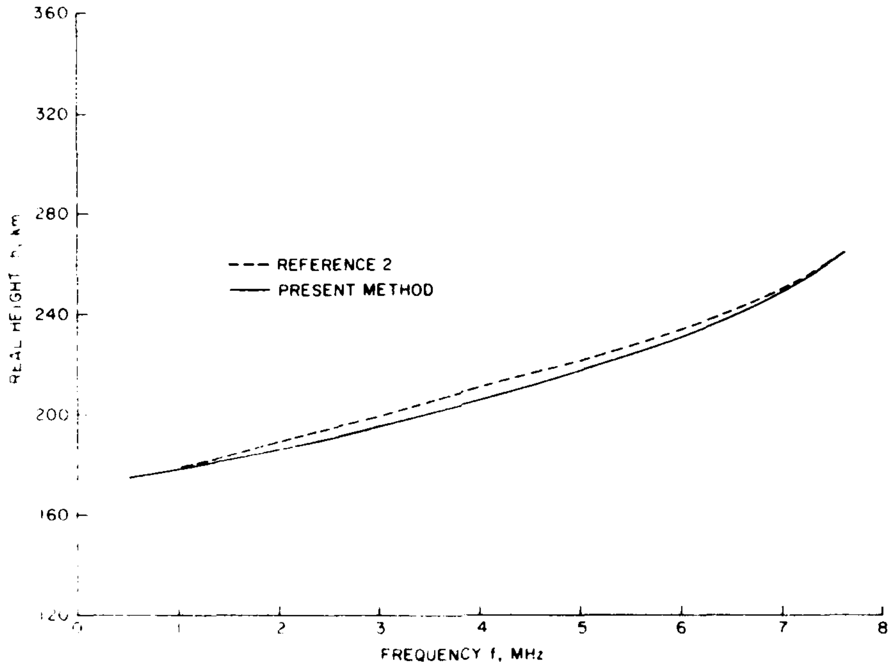


Figure 4. Comparison of Real Height by Present Method for Boulder Ionogram With Previous Calculation

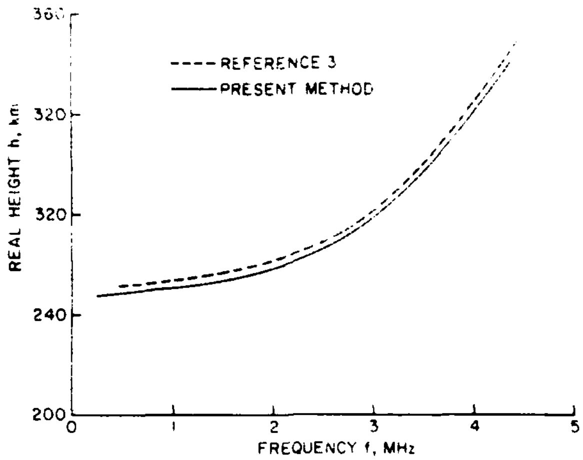


Figure 5. Comparison of Real Height by Present Method for Garchy Ionogram With Previous Calculation

5. REPRESENTATION OF VIRTUAL HEIGHT DATA IN SIMPLE PARAMETRIC FORM

Although the calculations of the increment in real height is simple in principle, it does require the numerical evaluation of an integral for each probing frequency. We have therefore explored the possibility of representing the virtual height by a simple parametric model from which the real height can be rapidly determined. Calculations over the range of data considered here showed that a one-parameter model was sufficient to represent the data with good accuracy. However, in going from the lower range of frequencies to the higher range, it was found necessary to change the parametric form in a simple way in order to maintain the same accuracy.

Since the calculations will be performed for several values of the probing frequency f_v, it is found advantageous to utilize the normalized probing frequency $x = f/f_v$. Plots of the increment in virtual height for two values of f_v are presented in Figures 6 and 7. The plots have been normalized to the maximum increment in virtual height. For convenience in performing the calculations, since the low frequencies contribute very little to the calculated height, the curves have been extrapolated to zero excess height at zero frequency. It may be noted at this point that the higher frequency curves show considerably more curvature than those at the lower frequencies. It is this increase of curvature with frequency that necessitates the change in parametric form in going to the higher frequencies.

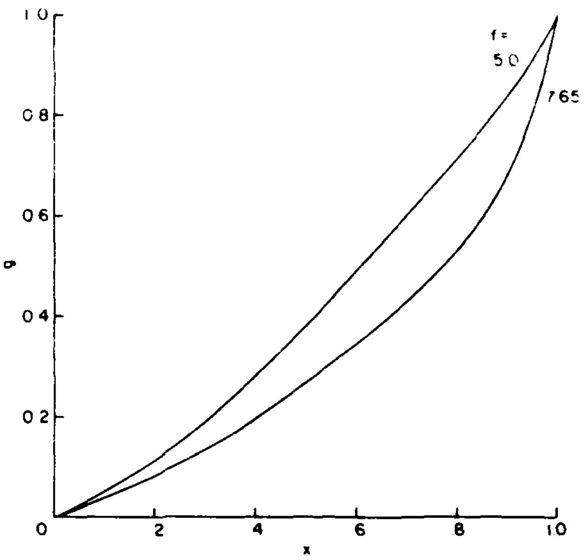


Figure 6. Plot of Virtual Height Function g Against Frequency x for Boulder Ionogram; the Function g is Normalized to Maximum Virtual Height

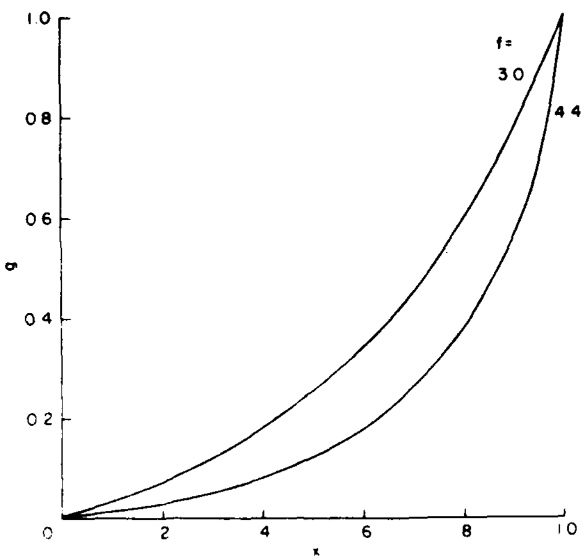


Figure 7. Plot of Virtual Height Function g Against Frequency x for Garchy Ionogram; the Function g is Normalized to Maximum Virtual Height

In looking for a possible parametric fit for these curves, we note that they are approximately linear in the lower range and then bend up rapidly as the frequency increases. We have, therefore, tried a polynomial form consisting of a linear term and a higher order term, of the form

$$g = \frac{h' - h'_0}{h'_{max} - h'_0} = a_1 x + a_2 x^n \ , \tag{16}$$

where h'_{max} is the maximum value of the virtual height and h'_0 is the extrapolated value of h' to zero frequency. A numerical study indicated that the data in the lower range of frequencies could be fitted by the polynomial

$$g_1 = ax + (1 - a)x^4 \tag{17}$$

while for the higher range of frequencies the form

$$g_2 = bx + (1 - b)x^6 \tag{18}$$

was a good model for the data. Plots of Eqs. (17) and (18) for several values of the governing parameter a or b are given in Figures 8 and 9. To show the kind of fit obtained, several data points from Figures 6 and 7 are replotted on these figures. A comparison of the ionogram data and the model curves indicated that the former could be represented by the parametric values:

Garchy, $f_v = 3.0$, $a = 0.45$

Garchy, $f_v = 4.4$, $b = 0.2$

Boulder, $f_v = 5.0$, $a = 0.75$

Boulder, $f_v = 7.65$, $b = 0.5$.

Utilizing Eq. (2) and performing the required integrations we obtain for the real height h,

lower frequencies, Eq. (17)

$$h - h_0 = \bar{p}(h'_{max} - h'_0)[aI_0 + (1 - a)I_1] \tag{19}$$

and higher frequencies, Eq. (18),

$$h - h_0 = \bar{p}(h'_{max} - h'_0)[bI_0 + (1 - b)I_2] \ , \tag{20}$$

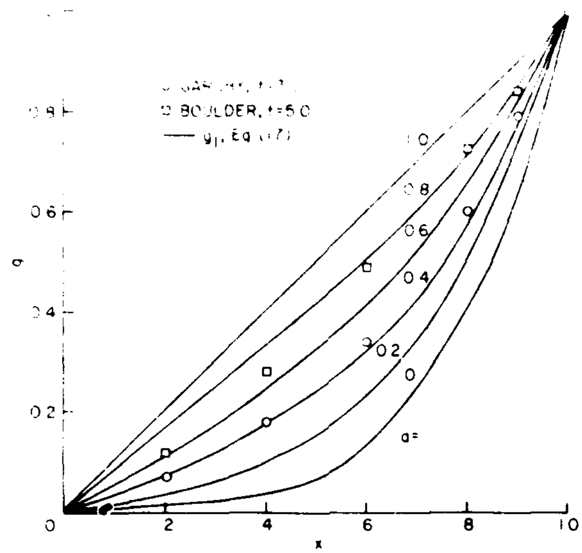


Figure 8. Virtual Height Function g for Lower Frequencies (Eq. 17) as a Function of Frequency for Several Values of Parameter a; Data Points at the Lower Frequencies From Both Ionograms are Also Plotted on the Figure

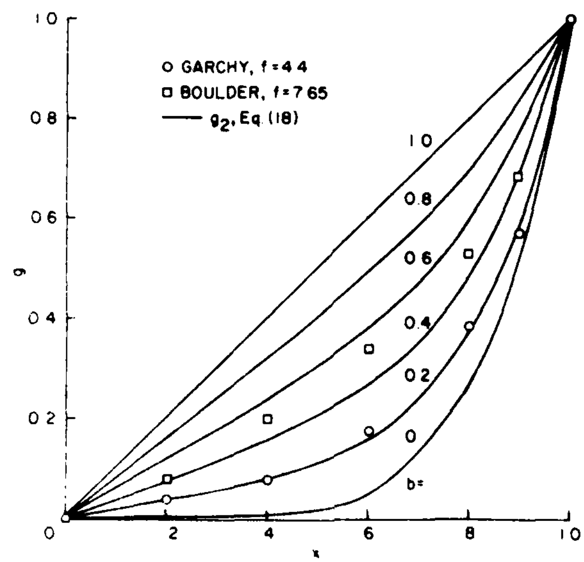


Figure 9. Virtual Height Function g for Higher Frequencies (Eq. 18) as a Function of Frequency for Several Values of Parameter b; Data Points at the Higher Frequencies From Both Ionograms are Also Plotted on the Figure

where $\bar{p}$ is an average value of p. The I integrals are given by

$$I_0 = \int_0^1 \frac{x^{\nu}}{(1 - x^2)^{\nu/2}} \, dx = \frac{1}{\pi^{1/2}} \Gamma\left(1 - \frac{\nu}{2}\right) \Gamma\left(\frac{1+\nu}{2}\right) \tag{21}$$

$$I_1 = \int_0^1 \frac{x^{\nu+3}}{(1 - x^2)^{\nu/2}} \, dx = \frac{1}{4} \Gamma\left(1 - \frac{\nu}{2}\right) \Gamma\left(2 + \frac{\nu}{2}\right) \tag{22}$$

$$I_2 = \int_0^1 \frac{x^{\nu+5}}{(1 - x^2)^{\nu/2}} \, dx = \frac{1}{12} \Gamma\left(1 - \frac{\nu}{2}\right) \Gamma\left(3 + \frac{\nu}{2}\right) . \tag{23}$$

Evaluation of these equations yielded the real heights:

Garchy, $f_v = 3.0$, $h = 284$ km

Garchy, $f_v = 4.4$, $h = 348$

Boulder, $f_v = 5.0$, $h = 215$

Boulder, $f_v = 7.65$, $h = 259$.

As seen from Figures 5 and 6, these values are in good agreement with those obtained by numerical integration. For the lower-frequency Boulder case, Figure 8, the choice of a proper value of the parameter a is not as clear cut as for the other cases. However, this need not concern us greatly since, as shown by an incremental analysis, a change of 0.05 in the parameter a results in a change of only about 1 km in the real height. The parametric procedure presented here thus appears to be a useful alternative to numerical integration in the determination of real height by the present method.

6. SUMMARY AND CONCLUDING REMARKS

A direct method for determining real height from virtual height data has been applied to two ionograms, corresponding to data obtained at Boulder, Colorado, and Garchy, France. The initial height was obtained by adapting the present method to the one-start procedure developed by Howe and McKinnis for this problem. The results obtained for the two ionograms were in good agreement with those obtained from previously-reported accurate numerical calculations.

To obviate the numerical integration required and thus make the present method more rapid and convenient, a procedure based on the modeling of the virtual height data in terms of a polynomial governed by a single parameter has been developed. The parameter may be determined by fitting the model to the given data. The results obtained with the model were in good agreement with those previously obtained by direct use of the data.

The present method affords a simple and convenient means of obtaining real heights from virtual height data, within the accuracy of the experimental data.

END

FILMED

8-83

DTIC